Surface Array Waveguide Mode Analyzer

J. M. Baird, D. H. Roper, and R. W. Grow

Microwave Device and Physical Electronics Laboratory
Department of Electrical Engineering
University of Utah

Abstract

A new type mode analyzer based on an array of field probes at the surface of an overmoded circular waveguide is described. This type device is needed in high power microwave research where waveguide sizes typically permit dozens of modes to propagate. Initial experiments at 11 GHz using a 2 inch diameter waveguide with 16 propagating modes and 20 surface probes have demonstrated the feasibility of the technique which relies on singular value decomposition algorithms for modal analysis. To make calibration feasible, the field probes must respond only to the E-field or the H-field at the waveguide wall. Optimization of probe placement, and anticipated errors are analyzed.

I. Introduction

A continuing problem in high power microwave (HPM) research is to measure the power content of higher order modes in large, over-moded waveguide structures. This paper describes the first analysis and implementation of a surface array waveguide mode analyzer (SAWMA) which may help solve this problem.

Efficient generation and utilization of HPM usually requires single mode operation in waveguides which are often large enough to propagate many dozens of modes. It is therefore critically important to be able to measure the effectiveness of HPM components in generating, controlling and converting waveguide modes. This is a difficult measurement, however, and the techniques typically used rely simply on deducing the relative modal content from thermal or radiation patterns. The most successful device to date is the k-space analyzer which spatially analyzes the RF energy radiated from a circular waveguide through a uniform array of small holes cut along the length of the guide. [1]

The concept of the SAWMA (surface array waveguide mode analyzer) is illustrated in Figure 1. The surface array consists of coupling elements which are inserted through small holes in the waveguide wall at selected positions to sample the RF fields at the waveguide wall. Two types of elements may be used: an E-field probe which samples the normal electric field, and/or an H-field probe which samples the tangential components of the magnetic field. The data from this array can then be used to solve for the amplitudes of the waveguide modes using modal analysis and singular value decomposition (SVD) techniques.

In this paper we show that this concept has the potential to identify the mode amplitudes to within a few percent accuracy even when several small amplitude modes exist in the presence of a large amplitude mode. The analysis described in section II shows that the concept works well to identify right and left circularly polarized modes as well as forward and backward propagating modes so long as the number of elements in the array exceeds the number of modes carrying power in the waveguide and which are included in the analysis. A significant advantage of the technique is that modes which are known not to carry power may be excluded from the analysis at any time thus limiting the number of array elements required in most cases.

It is critically important to optimize the placement of the probes on the surface of the circular waveguide. It has been found that satisfactory results can only be obtained when the system matrix which relates the measured fields to the waveguide modes has the smallest possible condition number. Section IV describes the procedure used to find the array pattern shown in Figure 2 which was used for the experimental SAWMA described in Section V. Analysis indicates that if the measurement errors can be reduced to the order of one percent, the errors in the significant mode amplitudes will be of the order of a few percent.

II. Theory of Operation.

The operation of the SAWMA technique can be illustrated by examination of the total fields which result from the superposition of a number of waveguide modes in a circular waveguide. Using transverse modal basis vectors $\mathbf{e}(\rho,\phi)$ and $\mathbf{h}(\rho,\phi)$ [2] we can write

$$\mathbf{E}_{\perp}(\rho,\phi,z) = \sum_{n=1}^{N} V_n(z)\,\mathbf{e}_n(\rho,\phi)$$
$$\mathbf{H}_{\perp}(\rho,\phi,z) = \sum_{n=1}^{N} I_n(z)\,\mathbf{h}_n(\rho,\phi) \qquad (1)$$

The basis vectors in this formulation are derived from TE^z and TM^z vector potential theory and the z components of the fields are proportional to the vector potentials. When the fields are evaluated at the waveguide radius $\rho = a$, only the E_ρ, H_ϕ, and H_z components remain.

In the form shown in equation (1), each term represents a separate circular waveguide mode so that the single index n represents not only the usual index pair (m,n) in the (ϕ,ρ) directions but also separate TE and TM modes, separate even and odd degenerate modes and separate forward and backward wave modes. In this formulation, $V_n(z)$ carries the mode amplitude and the propagation phase for each mode, and the basis vector $\mathbf{e}_n(\rho,\phi)$ carries the transverse electric field variation. $I_n(z)$ and $\mathbf{h}(\rho,\phi)$ are then given by

$$I_n(z) = \frac{V_n(z)}{Z_n} \quad ; \quad \mathbf{h}_n = \hat{\mathbf{z}} \times \mathbf{e}_n$$

where Z_n is the wave impedance of the nth mode.

When the electric field in equation (1) is evaluated at the kth probe position. $(\rho,\phi,z) = (a,\phi_k,z_k)$, the result is a set of equations of the form

$$E_{\rho k} = \sum_{n=1}^{N} M_{kn} V_{0n} \qquad (2)$$

where $k = 1,2, ..., K$ gives one equation for each of the K probes and V_{0n} represents the amplitude of the nth mode. Written in matrix form, this becomes

$$\mathbf{E}_{meas.} = \mathbf{M}\,\mathbf{V}_{modes} \qquad (3)$$

where the vector $\mathbf{E}_{meas.}$ contains the values of $E_{\rho k}$ at the waveguide wall, and the vector $\mathbf{V}_{modes}$ contains the amplitudes of the individual waveguide modes. The K x N matrix **M** relates the K measured values of the E_ρ-field to the N waveguide mode amplitudes, V_{0n}.

When the number of probe positions in the surface array is greater than or equal to the number of modes included in the analysis ($K \geq N$), the linear system in equation (3) can be solved in the least squares sense to give the mode amplitudes. We have found that the success of this procedure is strongly dependent on the methods used for solution and on the condition number of the matrix **M**.

We have used the SVD (singular value decomposition) algorithm which not only provides the best solution results but also provides techniques for ameliorating the errors caused by near-singularity. [3] This is necessary because the condition number of **M** changes as a function of frequency and becomes very large near the cutoff for any given mode. Although we have minimized the condition number between cutoff frequencies, it is necessary to identify and eliminate the errors due to singularities near the cutoff frequencies. SVD provides the technique to accomplish this.

Note that the E-field SAWMA technique described relies on a measurement of the total electric field at the waveguide wall. This can only be accomplished using an E-field probe which detects the E_ρ field component and is insensitive to the H-fields. However, the TE_{0n} circular modes have no E_ρ component and cannot be detected by these probes. It is therefore necessary to also examine H-field probes.

From equations (1) it is immediately evident that matrix systems like those shown in equation (3) can also be devised for probes which respond to H_ϕ and/or H_z at the waveguide wall. It is also evident that mixtures of E-field and H-field probes can be used in such a matrix system. It is therefore necessary to identify the best combinations for implementation of a SAWMA.

By performing SVD analysis on simulated SAWMA data, we have determined that all of the circular waveguide modes can be measured if we use an array of E-field probes plus one H_z-field probe for each TE_{0n} mode in the system. We have also found, however, that an array of H-field probes alone can provide equivalent results. To do this, a mixture of H_ϕ and H_z directions need to be measured, but conceivably this is simply a matter of probe orientation. Because probe uniformity in the array is desirable from a fabrication point of view, we are currently working toward developing a SAWMA based on a single type of H-field probe. Such a probe is described further in the next section.

We conclude this section by noting that there are several different ways to identify the mode amplitudes and the matrix elements M_{kn} depending on where the natural parameters in the system are included. We have found, however, that the normal mode formulation given in equations (1) has advantages. When the mode amplitudes are determined in this way not only is it easy to determine the power in each mode as 1/2 Re(VI*), but the normalization involved improves the conditioning of the matrix **M**.

III. Probe Design

To implement the SAWMA algorithms described above it is necessary to devise probes which are sensitive either to the E-field or to the H-field alone. When power is coupled to the probe this cannot be accomplished in the strictest sense, but we have devised two probes for which it is approximately true. We note that it is theoretically possible to devise a SAWMA using any type probe but calibration would require measurement of the scattering matrix of the full system of probes which is practically impossible.

The simplest type probe for use in a SAWMA is an E-field probe formed by putting a coaxial cable through the waveguide wall and letting the center wire end near the surface of the waveguide. This provides a probe which couples at the -30 to -50 dB level and is insensitive to the H-field. We have demonstrated the insensitivity to the H-field both by experiments in non-overmoded waveguides (no coupling where the normal E-field is zero) and by numerical simulations of probes in an overmoded circular waveguide. The numerical simulations were accomplished using a finite difference time domain (FDTD) computer code with which we were able to reproduce the measured probe results. [4]

It appears that an H-field probe which is insensitive to the normal E-field can be obtained by using a small cutoff slot in the waveguide wall. Symmetry requires that the normal E-field at the wall can only excite higher order cutoff modes which decay away much more rapidly than the fundamental slot mode. Thus, if coupling is accomplished after several e-folding lengths of the fundamental slot mode, the coupling depends only on the H-field at the waveguide wall. An H-field probe based on this concept has been designed, but it has not yet been measured.

One of the advantages of an H-field probe is that it can detect all types of waveguide modes because all modes have H-field components at the wall. Therefore, only one type probe need be used in a SAWMA. Another advantage is that the slot in the waveguide wall can be naturally adapted to excite a balanced line mode in a microstrip configuration. Then, with the probe signal coupled to standard microstrip circuits, it is possible to use strandard signal processing and detection at each probe position. This makes it feasible to achieve paralleled data acquisition which will be required for pulsed SAWMA operation.

IV. Probe Placement, and Error Analysis.

In the theory of linear algebra, the fractional error in the norm of the solution vector for a linear system of equations is proportional to the condition number of the matrix.[5] The condition number of the matrix **M** used in the SAWMA algorithm is determined at any given frequency by the array pattern used. If the vectors in the matrix were orthogonal, the condition number would be at its lowest value (unity), but because of the nature of the waveguide mode fields, this is not possible. We have therefore experimented numerically with a great many different probe patterns in an effort to find a best overall solution for an X-band SAWMA covering 8 to 12 GHz in a 2 inch diameter waveguide. Condition numbers as low as 50 have been observed.

Using numerical calculations of the SAWMA algorithm we have determined that the best probe patterns seem to be random. In these calculations we first select an array pattern and calculate the resulting matrix **M** for a given frequency. Then we assume a set of mode amplitudes and calculate the expected field amplitudes $E_{\rho k}$, to effectively obtain a set of ideal data. To simulate the errors in measured data, we then corrupt the field amplitudes with random errors on the order of 1 to 10 percent. These data are then used to solve the matrix system for the mode amplitudes using the SVD algorithm and the results are compared with the original mode amplitudes. In every case which included any regularity in the probe pattern, the matrix condition number and the mode amplitude errors were worse than those obtained using a good random probe pattern such as that shown in Figure 2.

Typical errors for a good 21 probe array pattern are shown in Table 1 This case was run at 10 GHz with 16 propagating modes assumed to have amplitudes and phases as shown in the first table column. After corrupting the ideal data with 1% random errors and using SVD to invert the system, the results are shown in the second column. The errors are shown in the third column. Although the errors for some of the small modes are quite high, this is to be expected when the system is carrying a large amplitude mode. Note that the errors in the 5 to 10 percent level modes are quite acceptable. (see box in table)

To illustrate the need for the SVD algorithm, when this same system was inverted using the standard least squares method of multiplying the matrix equation by the transpose, $\mathbf{M}^T$, to square the system before solution, the error in the 10 percent mode was 150 percent and that of the 5 percent mode was 163 percent. The reason for this was that the condition number for the square matrix $\mathbf{M}^T\mathbf{M}$ was about 37000 due to the close proximity of two of the modes to cutoff.

We have also experimented numerically with optimization of the probe positions. To do this, we started with a good random pattern for 20 probes which was close to that shown in Figure 2. We then used Gramm-Schmidt orthogonalization on **M** to obtain a new **M'** whose rows were orthogonal. This was followed by using Newton's optimization method to minimize the difference between $\mathbf{M}(\phi,z)$ and **M'** using the two position parameters (ϕ_k,z_k) for each row. The resulting new probe placements differed from the original placements by only a small amount, but the condition number changed from 105 to 80. The resulting best overall array pattern is shown in Figure 2, and this pattern was used for fabrication of the SAWMA described in the next section. The best condition number yet observed is 59 at 11 Ghz.

V. Initial Experimental Evaluation.

Figure 3 shows a photograph of the X-band SAWMA which was implemented in 2 inch diameter waveguide using 20 probes of the E-field type which were placed as shown in Figure 2. The intended frequency range is 8 to 12 GHz and calculations indicate that this SAWMA design provides good overall operation for modes in this frequency range. At the high frequency end, there are 10 different types of modes which propagate and the size of the probe array was determined on the basis of measuring up to 20 forward wave modes including all of the even and odd degenerate modes.

At the present time a few experiments have been carried out using the transmission mode of the HP8510 network analyzer to measure the relative amplitudes and phases of the signals at each probe position. This was done at 11 GHz with the SAWMA fed in two different ways. First, a long gradual taper (about 20 inches) was used to make a transition from WR-90 rectangular waveguide to the 2 inch diameter SAWMA waveguide. This was intended to create a pure TE_{11} mode for measurement. The results are shown in Figure 4. Note that there is some indication of a small amount of the TE_{12} mode. This may be an actual result since a long gradual taper does couple slightly to this mode.

After this initial experiment, a section of the circular taper was removed to create a step in the waveguide diameter. An additional length of 2 inch diameter waveguide was inserted between the step transition and the SAWMA to prevent evanescent modes from interfering with the measurements. The step transition used is analytic and can be shown to excite TM_{11} and TE_{12} modes with the expected amplitudes shown in Figure 5. The results of three different SAWMA measurements taken at different times and after some disassembly of the system are also shown. In each of these tests, the data shown are the total amplitudes of the combined even and odd modes of each type.

Since the waveguide step was fed with a TE_{11} linearly polarized wave, we expect that the modes of each type will also be linearly polarized. For the three significant modes in Figure 5, Figure 6 shows a rectangular coordinate plot of the time evolution of the total E-field vectors based on the measured amplitudes of the even and odd modes. The results show that the measured modes are very nearly linearly polarized as expected. The slight tilt angle of the linear polarization is simply the angle between the actual polarization and the reference angle of the surface array. For the small measured modes, polarization is nearly random which is consistent with the fact that excitation of these modes is primarily due to random errors.

A noise averaging mode of operation in the 8510 network analyzer was used to take the data shown in Figures 4 to 6.

VI. Summary and Conclusions.

The technique of using a surface array of waveguide field detectors to determine the mode content in overmoded waveguide systems shows considerable promise for use in high power microwave research. The technique will handle dozens of modes with reasonable accuracy and it is relatively easy to devise probes which are compatible with high field strength. Initial experiments with an X-band SAWMA based on E-field probes shows promising results, and work is continuing to further characterize this device and to build a second device based on H-field probes.

This work was supported by Harry Diamond Laboratories through EG&G, Inc./Idaho National Engineering Laborat . .

[1] W. Kasparek, G. A. Muller, "The wavenumber spectrometer--an alternative to the directional coupler for multimode analysis in oversized waveguides," *Int. J. Electronics*, Vol. 64, pp. 5-20, 1988.

[2] R. F. Harrington, *Time Harmonic Electromagnetic Fields*, McGraw-Hill, New York, 1961, Chapter 8.

[3] W. H. Press, et al., *Numerical Recipies*, Cambridge Univ. Press, New York, 1987.

[4] D. H. Roper, J. M. Baird, "Analysis of Overmoded Waveguides Using the Finite Difference Time Domain Method," in this digest. (1992 IEEE International Microwave Symosium MTT-S.)

[5] R. L. Burden, J. D. Faires, *Numerical Analysis*, PWS-KENT Publishing Co. Boston, 1989, pp. 416-421.

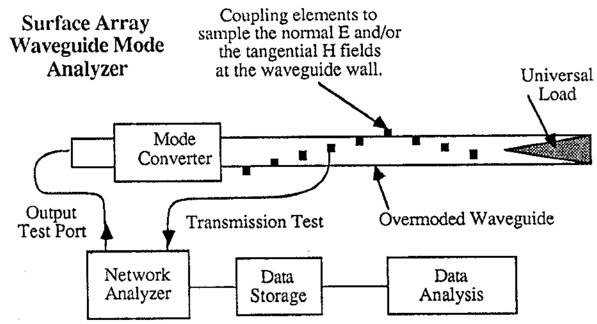

Figure 1. Conceptual diagram of surface array waveguide mode analyzer based on analyzing RF transmission to spatially distributed waveguide coupling elements

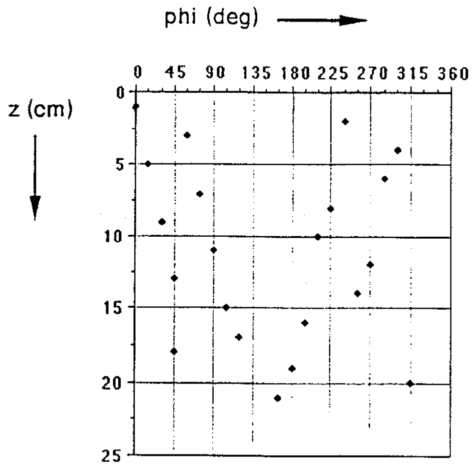

Figure 2. Positions of array elements for X-band SAWMA.

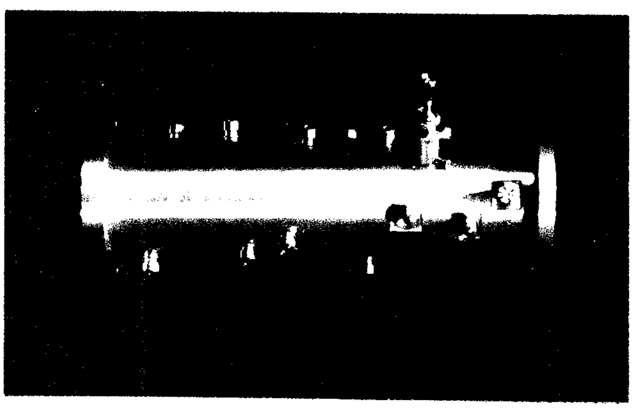

Figure 3. Photograph of X-band SAWMA.

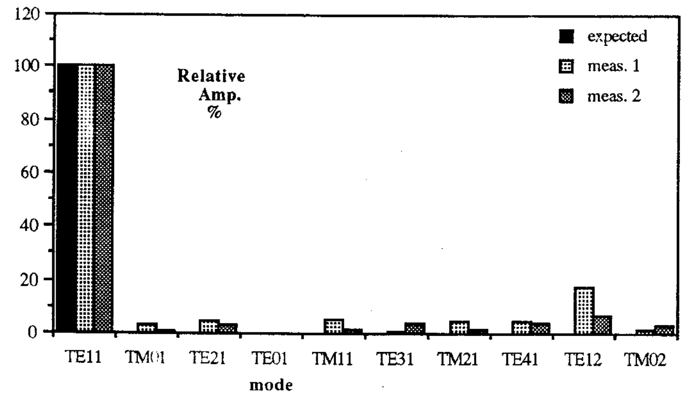

Figure 4. Measured results for a "pure" TE_{11} mode.

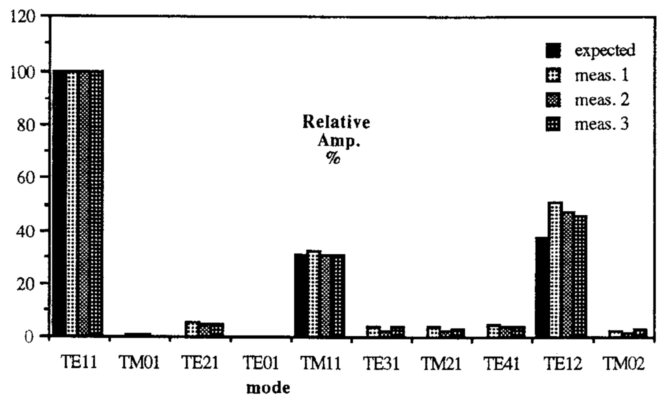

Figure 5. Comparison of modes in stepped waveguide.

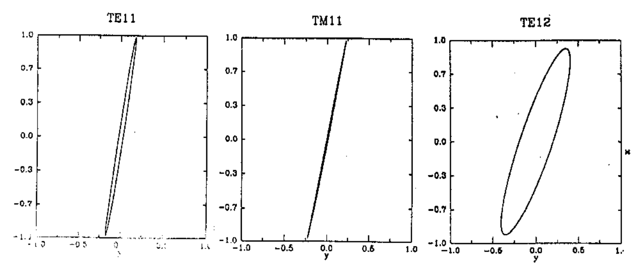

Figure 6. Spatial polarization for the three modes.

mode	Coeff. (mag,phi)		Calc. (mag,phi)		ERROR: mag	phase
1	100.0000,	.0000	99.4152,	.0492	.5848 %	.0492 deg
	100.0000,	90.0000	99.8759,	90.0120	.1241 %	.0120 deg
2	7.0000,	.0000	7.0650,	-1.3283	.9286 %	1.3283 deg
	.0000,	.0000	.0000,	.0000	----	----
3	3.5355,	44.9999	2.5228,	39.1247	28.6452 %	5.8752 deg
	3.2655,	69.5670	3.0458,	68.4796	6.7257 %	1.0874 deg
4	.0000,	.0000	.0000,	.0000	----	----
	5.0000,	.0000	5.3181,	.6641	6.3611 %	.6641 deg
5	10.0000,	.0000	9.6504,	-1.4442	3.4963 %	1.4442 deg
	3.0000,	90.0000	3.2228,	86.1383	7.4272 %	3.8617 deg
6	5.0000,	.0000	4.9313,	2.7556	1.3740 %	2.7556 deg
	5.0000,	.0000	4.9671,	-2.9585	.6576 %	2.9585 deg
7	2.0000,	90.0000	5.0100,	89.0243	150.5013 %	.9757 deg
	2.0000,	90.0000	3.6334,	-169.3281	81.6723 %	100.6719 deg
8	1.0000,	.0000	1.4172,	-15.3910	41.7226 %	15.3910 deg
	1.4142,	44.9999	1.7791,	43.1003	25.7991 %	1.8996 deg

Table 1. Typical solution results using SVD.